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Constrained Output Feedback H_∞ Control of a Four-tank System

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***Abstract*— In this paper, control of liquid level for a four-tank system with respect to disturbances is studied, where constrained output feedback H_∞ control of linear systems based on linear matrix inequality is adopted. In contrast, both an unconstrained H_∞ controller and decentralized PID controllers are applied to the four-tank system. The simulation results illustrate the effectiveness of constrained output feedback H_∞ control strategy.**

***Index Terms*— Four-tank system; H_∞ control; Output feedback; Constrained systems.**

I. INTRODUCTION

The recent investigation shows that a large quantity of industrial processes have the common characteristic such as strong coupling, nonlinearity, time delay and easily being disturbed etc. Four-tank systems are designed to simulate complicated industrial control systems, which display complex dynamics such as zero transmission, multiply loops interaction, non-minimum phase [1].

PID control is a powerful method in industrial application owing to following advantages: great operating flexibility, simple design and tuning [2]. However PID control cannot be applied to multi-input multi-output (MIMO) systems directly. Undesired interaction in different loops can be eliminated by decoupling control method. Therefore, decoupled PID control technique, which is regarded as an effective method, can be used in four-tank systems [3]–[5]. While a system becomes sensitive to parameter changing, and not all MIMO systems satisfy decouple conditions. Adaptive neuro fuzzy inference system (ANFIS) method is carried out in [6]. Furthermore, multivariable internal model control (IMC) is applied in [2], in which the controller successfully stabilize system at the setpoint and disturbances can be attenuated. Also, model predictive control (MPC) technology can handle both loop interaction and constraints [7]. In [8], an experimental study of nonlinear model predictive control (NMPC) is presented on a four-tank system, where inputs and state constraints are sufficiently considered. The result shows that stability constraints in NMPC are necessary in order to guarantee closed-loop stability. Distributed model predictive control (DMPC) framework is applied in an experimental four-tank system [9], which provides significant improvement over completely decentralized MPC controllers. Besides, H_∞ control method is to design a controller which can achieve the stabilization of the closed-loop system and reduce the impact of disturbances on the desired performance with a set of finite energy disturbance signals [12]. Thus H_∞ control method is chosen in this paper.

The voltage of pumps of a four-tank system are not allowed to exceed the limit value which means control input constraints in this system. In some unexpected scenarios, the liquid flows out of tank through a leaky hole. Thus, a controller has to have the ability to maintain the liquid level. A control problem of three-tank system based on H_∞ theory with leaky disturbance is proposed in [13], where the system can achieve disturbance rejection and satisfy hard constraints. Output feedback H_∞ control of constrained linear systems is easily implemented in practice [12]. In this paper, problem of constrained output feedback H_∞ control for a four-tank system is considered, where a design procedure of output feedback H_∞ controllers is derived via the LMI scheme. Both unconstrained H_∞ and PID controllers are designed for comparisons.

This paper is expanded as follows: a four-tank system is described in Section II. Section III presents the output feedback control law and solutions. Both constrained output feedback H_∞ and unconstrained H_∞ controller are designed for a four-tank system in Section IV. Section V shows two groups of simulation. Section VI concludes the paper with a short summary.

II. FOUR-TANK SYSTEM

In this section a mathematical model for the four-tank system is described. A schematic representation of the four-tank system is given in Fig. 1.

The four-tank system consists of four tanks, two pumps and four valves. Pump 1 supplies liquid to tank 1 and tank

4 , pump 2 supplies liquid to tank 2 and tank 3. The flow of pump 1 and pump 2 are split up by valve 1 and valve 2 respectively. There is a pipe under every tank making the liquid from an upper tank to flow into a lower tank. The liquid from tank 1 and tank 2 flows into the reservoir below. Valve 3 and valve 4 are designed in order to simulate leakage disturbances.

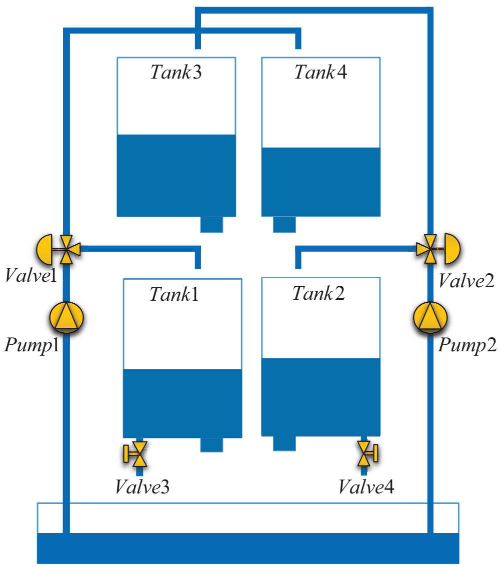


Fig. 1. Schematic diagram of the four tank system

TABLE I
PARAMETERS IN FOUR TANK MODEL

Model parameters	Values
cross-section of tanks (A_i)	$730cm^2$
cross-section of pipes (a_i)	$2.3cm^2$
cross-section of valve 3 and valve 4 (s)	$2.0cm^2$
liquid inflow rate (β_1)	0.333
liquid inflow rate (β_2)	0.307
flow coefficient (k_1)	$5.51cm^3/s$
flow coefficient (k_2)	$6.58cm^3/s$
gravitational acceleration (g)	$981cm/s^2$

Take liquid leakage into consideration, the dynamics of the four-tank system is described by [9]

$$
\begin{aligned}
\dot{h}_1 &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\beta_1 k_1}{A_1}v_1 - \frac{s\sqrt{2gh_1}}{A_1}\omega_1, \\
\dot{h}_2 &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\beta_2 k_2}{A_2}v_2 - \frac{s\sqrt{2gh_2}}{A_2}\omega_2, \\
\dot{h}_3 &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\beta_2)k_2}{A_3}v_2, \\
\dot{h}_4 &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\beta_1)k_1}{A_4}v_1,
\end{aligned} \tag{1}
$$

where ω_i $(i=1,2)$ represents the percentage of the opening of valve 3 and valve 4, h_i $(i=1,\ldots,4)$ denotes the liquid level of the tank, v_i $(i=1,2)$ indicates the percentage of the pump's voltage which determines the flow of the pump. Therefore, the maximum value of v_i is 1. The parameters A_i, a_i, β_i, k_i, s and g, and its values are given in Table I.

III. CONSTRAINED OUTPUT FEEDBACK H_∞ CONTROL OF LINEAR SYSTEMS

In this section, a constrained output feedback H_∞ controller of constrained linear systems is designed. Consider the following linear time-invariant (LTI) system

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t), \\
z_1(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t), \\
z_2(t) &= C_2 x(t) + D_{21} w(t) + D_{22} u(t), \\
y(t) &= C_3 x(t) + D_{31} w(t),
\end{aligned} \tag{2}
$$

subject to output constraints

$$
|z_{2i}(t)| \le z_{2i,\max}, \quad i = 1,2,\ldots,n_{z2}, \quad t \ge 0. \tag{3}
$$

Here $x \in R^{n_x}$ is the system state, $z_1 \in R^{n_{z1}}$ the performance output, $z_2 \in R^{n_{z2}}$ the constraint output, $y \in R^{n_y}$ the measurement output, $u \in R^{n_u}$ the control input, $w \in R^{n_w}$ the disturbance input. It is assumed that $D_{21} = 0$ and $D_{31} = 0$, i.e., disturbances have no direct way to affect the constrained outputs and the measured outputs.

Some assumptions are clarified as follow: (A, B_2, C_3) is stabilizable and observable, the disturbances energy are bounded in a compact set,

$$
\mathscr{W} := \left\{ w \in R^{n_w} \,\middle|\, \int_0^\infty \|w(\tau)\|_2^2 d\tau \le w_{max} \right\}. \tag{4}
$$

We consider the output feedback control law K,

$$
\begin{aligned}
\dot{\xi}(t) &= A_k \xi(t) + B_k y(t), \\
u(t) &= C_k \xi(t) + D_k y(t).
\end{aligned} \tag{5}
$$

where $\xi \in R^{nk}$ is the state of controller, A_k, B_k, C_k and D_k are appropriate constant matrices which need to be calculated.

Applying controller (5) to the LTI system (2). Thus, the closed-loop system is

$$
\begin{aligned}
\dot{x}_{cl}(t) &= A_{cl} x_{cl}(t) + B_{cl} w(t), \\
z_1(t) &= C_{cl,1} x_{cl}(t) + D_{cl,1} w(t), \\
z_2(t) &= C_{cl,2} x_{cl}(t) + D_{cl,2} w(t),
\end{aligned} \tag{6}
$$

where $x_{cl} = \begin{bmatrix} x \\ \xi \end{bmatrix}$, $A_{cl} = \begin{bmatrix} A + B_2 D_k C_3 & B_2 C_k \\ B_k C_3 & A_k \end{bmatrix}$, $B_{cl} = \begin{bmatrix} B_1 + B_2 D_k D_{31} \\ B_k D_{31} \end{bmatrix}$, $C_{cl,1} = \begin{bmatrix} C_1 + D_{12} D_k C_3 \\ D_{12} C_k \end{bmatrix}^T$, $D_{cl,1} = \begin{bmatrix} D_{11} + D_{12} D_k D_{31} \end{bmatrix}$, $C_{cl,2} = \begin{bmatrix} C_2 + D_{22} D_k C_3 & D_{22} C_k \end{bmatrix}$, $D_{cl,2} = \begin{bmatrix} D_{21} + D_{22} D_k D_{31} \end{bmatrix}$. The objective of a constrained output feedback H_∞ control is to guarantee the closed-loop system (6) internal stability, the H_∞ performance from the disturbance w to the performance output z_1 is minimized, while the output constraints z_2 are satisfied.

For a given scalar $\gamma > 0$, the H_∞ performance from $w(t)$ to $z_1(t)$ is less than γ, suppose that there exists a matrix such that $X_{cl} = X_{cl}^T > 0$ satisfying the following linear matrix inequality (LMI) [12]

$$\begin{bmatrix} A_{cl}^T X_{cl} + X_{cl} A_{cl} & X_{cl} B_{cl} & C_{cl,1}^T \\ * & -\gamma I & D_{cl,1}^T \\ * & * & -\gamma I \end{bmatrix} \le 0, \tag{7}$$

where $*$ replaces blocks that are readily inferred by symmetry.

Denote $V(x_{cl}) := x_{cl}^T X_{cl} x_{cl}$. The feasibility of (7) guarantees that for the closed-loop system (6) [15]

$$\frac{d}{dt} V(x_{cl}(t)) + \|z_1(t)\|^2 - \gamma^2 \|w(t)\|^2 \le 0. \tag{8}$$

The integration of (8) from 0 to t is

$$V(x_{cl}(t)) + \int_0^t \|z_1(\tau)\|^2 d\tau \le \gamma^2 \int_0^t \|w(\tau)\|^2 d\tau + V(x_{cl}(0)), \tag{9}$$

where $t \ge 0$. The compact set (4) and the inequality (9) imply that

$$\begin{aligned} \Omega(X_{cl}, \alpha) := \Big\{ & x_{cl} \in R^{n_x} \mid V(x_{cl}) \le \alpha, \\ & \alpha := \gamma^2 w_{max} + V(x_{cl}(0)) \Big\}. \end{aligned} \tag{10}$$

is an ellipsoid where the state trajectory stays in. Therefore α is an adjustable parameter of constrained output feedback H_∞ controller.

Suppose that there exists an optimal solution X, Y and $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$ to the following semi-definite programming as follow [12, Theorem 1]:

$$\min_{X>0, Y>0, \hat{A}, \hat{B}, \hat{C}, \hat{D}} \gamma \quad s.t. \; LMIs \; (12), (13) \tag{11}$$

$$\begin{bmatrix} S_0 & S_1 & B_1 & (C_1 X + D_{12}\hat{C})^T \\ \star & S_2 & Y B_1 & (C_1 X + D_{12}\hat{D} C_2)^T \\ \star & \star & -I & (D_{11} + D_{12}\hat{D} D_{31})^T \\ \star & \star & \star & -\gamma^2 I \end{bmatrix} \le 0 \tag{12}$$

$$\begin{bmatrix} \frac{1}{\alpha} Z & M_0 & M_1 \\ \star & X & I \\ \star & \star & Y \end{bmatrix} > 0 \; with \; Z_{ii} \le z_{2i,max}^2 , \tag{13}$$

where $\star$ represents the transpose of the element across the diagonal, $S_0 = AX + XA^T + B_2\hat{C} + (B_2\hat{C})^T$, $S_1 = \hat{A}^T + A + B_2\hat{D}C_3$, $S_2 = A^T Y + YA + \hat{B}C_2 + (\hat{B}C_2)^T$, $M_0 = C_2 X + D_{22}\hat{C}$, $M_1 = C_2 + D_{22}\hat{D}C_3$. LMIs (12) and (13) are derived as shown in [12]. The constrained output feedback controller (5) has the capability to attenuate disturbances for energy bounded disturbances, and to satisfy the time-domain hard constrains (3).

Suppose that $\gamma^*, X^*, Y^*, \hat{A}^*, \hat{B}^*, \hat{C}^*, \hat{D}^*$ is the optimal solution of LMI optimization problem. Then, one can find nonsingular matrices M and N to satisfy $MN^T = I - XY$, and define the controller by [15]

$$\begin{aligned} D_k^* &:= \hat{D}, \\ C_k^* &:= (\hat{C}^* - D_k C_3 X) M^{-T}, \\ B_k^* &:= N^{-1}(\hat{B}^* - Y B_2 D_k), \\ A_k^* &:= N^{-1}(\hat{A}^* - N B_k C_3 X - Y B_2 C_k M^T \\ &\quad - YAX - Y B_2 D_k C_3 X) M^{-T}. \end{aligned} \tag{14}$$

IV. Controllers Design

The purpose of this paper is to design a controller to make the liquid levels of tank 1 and tank 2 track the target values, i.e., h_{01} and h_{02}, and the four-tank system quickly reach a new equilibrium state in case of disturbances. In this section, a constrained output feedback H_∞ controller, is designed for a four-tank system.

We linearize the four-tank system at the equilibrium point $h_0 = (h_{01}, h_{02}, h_{03}, h_{04})^T$, $v_0 = (v_{01}, v_{02})^T$. Denote x_i and u_i as $x_i = (h_i - h_{0i}) / h_{0i} \, (i = 1,2,3,4)$, $u_i = (v_i - v_{0i}) / v_{0i} \, (i = 1,2)$. Hence, the matrices of state space equation of the system (1) with disturbances is

$$A = \begin{bmatrix} \frac{-a_1 g}{A_1\sqrt{2gh_{01}}} & 0 & \frac{a_3 g h_{03}}{h_{01} A_1 \sqrt{2gh_{03}}} & 0 \\ 0 & \frac{-a_2 g}{A_2\sqrt{2gh_{02}}} & 0 & \frac{a_4 g h_{04}}{h_{02} A_2 \sqrt{2gh_{04}}} \\ 0 & 0 & \frac{-a_3 g}{A_3\sqrt{2gh_{03}}} & 0 \\ 0 & 0 & 0 & \frac{-a_4 g}{A_4\sqrt{2gh_{04}}} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -\frac{gs}{A_1\sqrt{2gh_{01}}} & 0 \\ 0 & -\frac{gs}{A_2\sqrt{2gh_{02}}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} \frac{\gamma_1 k_1 v_{01}}{h_{01} A_1} & 0 \\ 0 & \frac{\gamma_2 k_2 v_{02}}{h_{02} A_2} \\ 0 & \frac{(1-\gamma_2) k_2 v_{02}}{h_{03} A_3} \\ \frac{(1-\gamma_1) k_1 v_{01}}{h_{04} A_4} & 0 \end{bmatrix}.$$

$z_1(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ and $z_2(t) = \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T$ are chosen as performance output and constraint output of the output feedback H_∞ control, respectively. Thus,
$C_1 = C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, $D_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,
D_{11}, D_{12}, D_{21}, D_{31}, and C_2 are zero matrices with appropriate dimensions. The output constraints are chosen as $z_{2i,\max} = (v_{0,\max} - v_{0i}) / v_{0i} \, (i = 1,2)$.

An unconstrained H_∞ controller [17] is designed for four-tank systems, where $z_1(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ is performance output, $y(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ is measurement output. Other parameter matrices are as follows:

$$D_{21} = D_{22} = D_{31} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, C_2 = C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

V. Simulations

In this section, two groups of simulation of the four-tank system are carried out to illustrate the effectiveness of the constrained output feedback H_∞ control method.

As described in the previous section, the liquid level of tank 1 and tank 2 at h_{01} and h_{02} must be stabilized in case of leakages. The equilibrium points, open degree of the leakage valves, parameters of the PID and constrained output feedback H_∞ controllers are shown in Table II.

TABLE II
Parameters in Simulations

	Simulation1	Simulation2
h_{01} (cm)	12.0	25.0
h_{02} (cm)	10.0	20.0
h_{03} (cm)	5.6	12.2
h_{04} (cm)	4.8	9.2
v_{01} (%)	60.9	84
v_{02} (%)	52.8	78
ω_1 (%)	[0, 25]	[0, 100]
ω_2 (%)	[0, 25]	[0, 100]
P_1	5.05	3.265
I_1	0.00001	0
D_1	1	5
P_2	5.15	3.625
I_2	0.000013	0
D_2	2.5	9
α	60	110

In simulation 1, PID control method is presented for comparison. In simulation 2, both unconstrained H_∞ and PID control methods are exhibited for comparisons.

A. Simulation 1

In this subsection, we try to simulate a small amount of leakage for a long time.

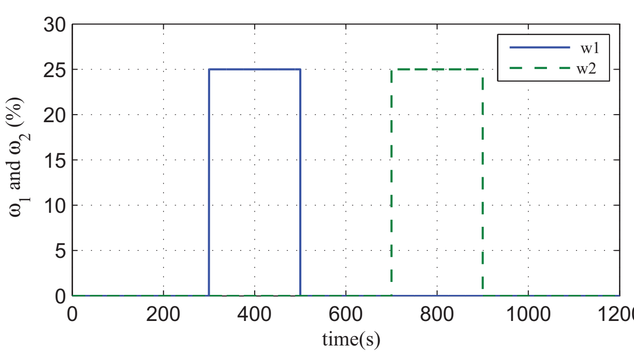
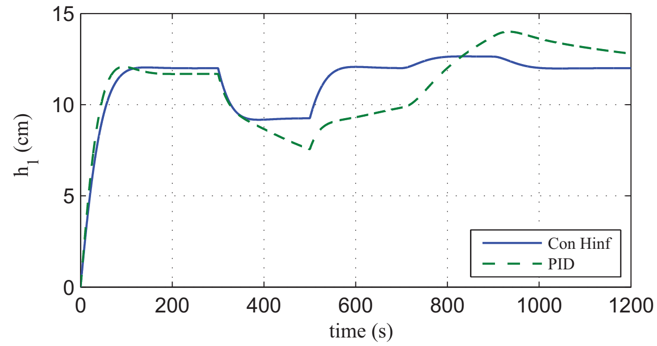
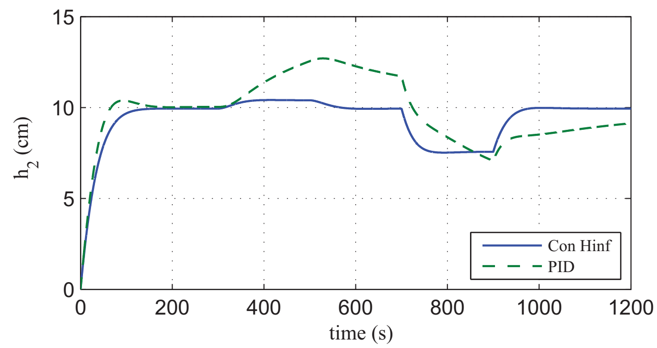


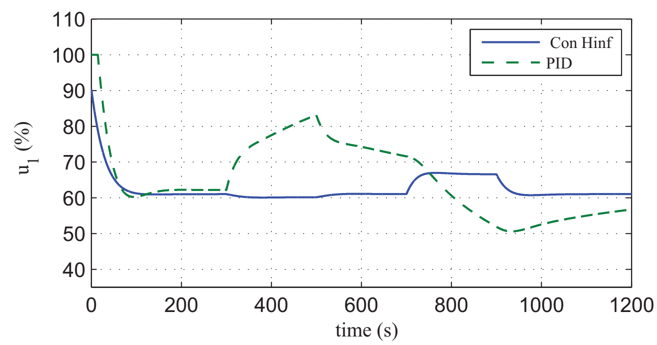
Fig. 2. Opening of leakage valves



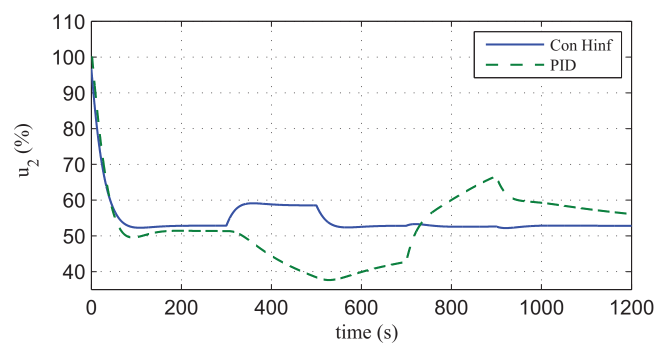
(a) Liquid level of tank 1.



(b) Liquid level of tank 2.



(c) Control input 1.



(d) Control input 2.

Fig. 3. Responses of the four-tank system in case of disturbances and specified input, where " Con Hinf" represents constrained output feedback H_∞ control.

Two decentralized PID controllers with control input saturations are considered for a reasonable comparison. Fig. 2 shows that the valves of leakage under tank 1 and tank 2 are opened at the time of 300 ~ 500s and 700 ~ 900s respectively.

According to Fig. 3(a) and Fig. 3(b), the four-tank system approaches to its equilibrium at 160s by constrained output feedback H_∞ method, while it reached equilibrium point at about 200s by PID controller. As Fig. 3 shows, the liquid levels of the tanks return to the expected set points in 200s after the leakages disappeared. The four-tank system quickly reaches steady state in the presence of disturbances. There is little interaction between different loops during the whole process.

However, undesired interaction can not be eliminated by PID control method. The liquid level of one tank suddenly increases when another tank leaks. The results show that the large fluctuations of liquid levels and control inputs occur when the disturbances appear. The liquid levels return its equilibrium of the four-tank system with PID controllers after a very long time, so that the curves are omitted.

The PID controllers are tuned separately. Thus tank 1 is compensated by pump 1 when it is leaking, tank 2 is compensated by pump 2 when it is leaking. The solid lines in Fig. 3(c) and Fig. 3(d) show that tank 1 is fed by pump 2, tank 2 is fed by pump 1 when the system controlled by constrained output feedback H_∞ method. The decoupled property makes the dynamics of the system more smoothly.

B. Simulation 2

In this subsection, an unconstrained H_∞ controller, two decentralized PID controllers without control input saturations are considered for a reasonable comparison. Fig. 4 shows that the valves of leakage under tank 1 and tank 2 are opened at the time of 450 ~ 500s and 2000 ~ 2050s respectively. This subsection is to simulate a large number of leakage for a short time.

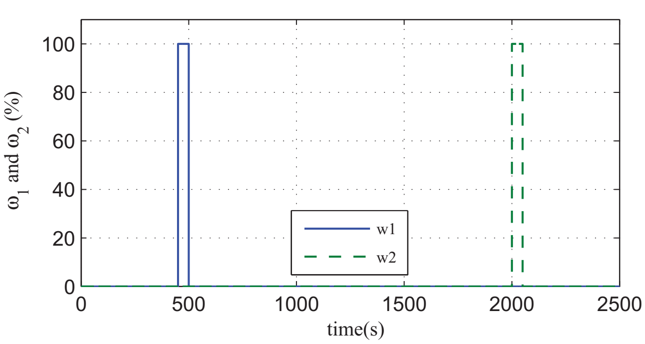
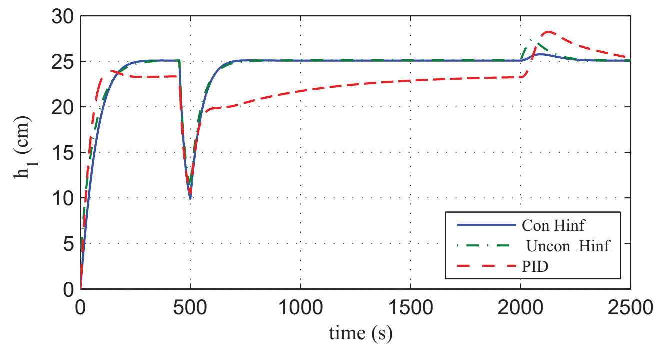
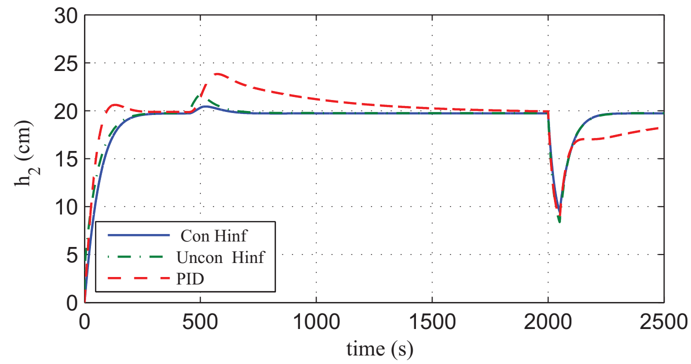


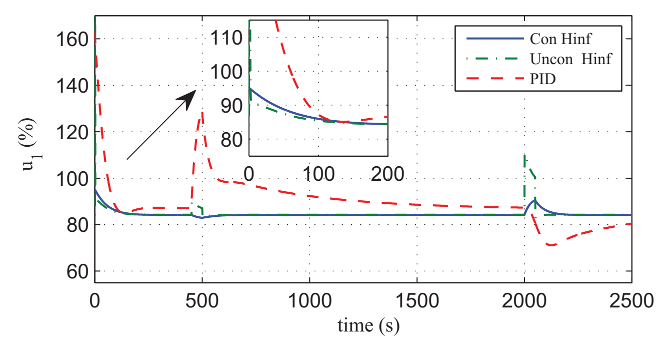
Fig. 4. Opening of disturbance valves



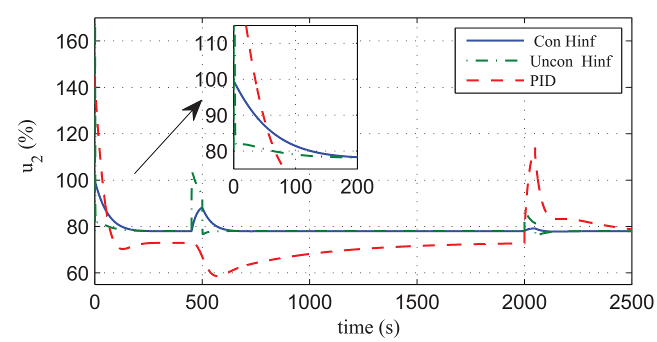
(a) Liquid level of tank 1.



(b) Liquid level of tank 2.



(c) Control input 1.



(d) Control input 2.

Fig. 5. Responses of the four-tank system in case of disturbances and specified input, where " Con Hinf" represents constrained output feedback H_∞ control, " Uncon Hinf" represents unconstrained output feedback H_∞ control.

As Fig. 5(a) and Fig. 5(b) show, the four-tank system reaches its equilibrium at about 750s by both constrained and unconstrained H_∞ controllers, while it reached equilibrium point at about 1900s by PID method, if tank 1 leaks.

The curves of tank 2 leaks are omitted simply because they are similar. The liquid level compensation capability of the three control methods are all the same when the liquid leakages occur. The ability of restraining unexpectedinteraction of constrained output feedback H_∞ controller is better than other two presented controllers. As Fig. 5(c) and Fig. 5(d) show, PID control and unconstrained H_∞ control method can not handle constraints, which result in the control inputs are out of 100% at the beginning and while tank 1 and tank 2 leak seriously.

VI. Summary

In this paper, the design procedure of a constrained output feedback H_∞ controller for a four-tank system was proposed. The effectiveness of the controller was shown in Section V, in which a small amount of leakage for a long time and a large number of leakage for a short time are considered, respectively. In contrast, both PID and unconstrained H_∞ controllers were presented.

The results shown that the constrained output feedback H_∞ controller could make the liquid levels of tank 1 and tank 2 track the target values and the four-tank system quickly reach a new equilibrium state in case of disturbances. In addition, the pump's voltage have no more than the maximum value from beginning to end. Therefore, constrained output feedback H_∞ method could attenuate disturbances, satisfy the time-domain hard constraints, guarantee the stability of closed-loop system.

Acknowledgment

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